Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Real Analysis

Subject Code: 4SC06RAC1 **Branch: B.Sc.(Mathematics)**

Semester : 6 **Date : 11/04/2017** **Time : 2:30 To 5:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: **(14)**

a) Define: p-series. (1)

b) Give one example of oscillating sequence. (1)

c) True/false: Any subsequence of convergent sequence is convergent. (1)

d) Define: Cauchy sequence. (1)

e) Is the limit point of sequence unique? (1)

f) True/false: For the series $\sum u_n$ if $\lim_{n\to\infty} u_n \neq 0$ then the series must be divergent. (1)

g) Give one example of oscillating sequence. (1)

h) Define: Infinite series. (1)

i) Write the range of the sequence $\{ (-1)^n \}$. (1)

j) Define : Darbouxe's lower sum of the function on [a,b] (1)

k) Write the statement of the comparison test for the convergence of series. (2)

l) State sandwich theorem for the sequence. (2)

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions **(14)**

a) State and prove Bolzeno-weiestrass theorem for sequence. **(7)**

b) Prove that A necessary and sufficient condition for the convergence of monotonic sequence that it is bounded. **(7)**

Q-3 Attempt all questions **(14)**

a) State and prove Cauchy's general principle of convergence of sequence. **(6)**

b) Using $\varepsilon - m\ definition$ prove that **(5)**

(1) $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$

(2) $\lim_{n\to\infty} \sqrt[n]{a} = 1$, where a > 0 .

c) Test the convergence for the series. **(3)**

(1) $\frac{1}{2} + \frac{1}{2.2^2} + \frac{1}{3.2^3} + \frac{1}{4.2^4} + \ldots$

(2) $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots$

Q-4 **Attempt all questions** **(14)**

a) Prove that the sequence $\{s_n\}$, where $s_n = \left(1+\frac{1}{n}\right)^n$, is convergent and $\lim \left(1+\frac{1}{n}\right)^n$ lies between 2 and 3. **(6)**

b) Show that the sequence $\left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots \ldots \ldots \frac{1}{(n+n)^2}\right]$ converges to zero. **(4)**

c) Find using Riemann sum $\int_{-1}^{1} |x| dx$. **(4)**

Q-5 **Attempt all questions** **(14)**

a) Prove that A positive term series $\sum \frac{1}{n^p}$ is convergent if and only if $p > 1$. **(8)**

b) State and prove D'Alembert Ratio test for the convergence of series. **(6)**

Q-6 **Attempt all questions** **(14)**

a) State and prove Leibnitz test for alternating series . **(8)**

b) If f is bounded and integrable on $[a, b]$, then show that $|f|$ is also bounded and integrable on $[a, b]$. Moreover $\left|\int_a^b f\, dx\right| < \int_a^b |f| dx$. **(6)**

Q-7 **Attempt all questions** **(14)**

a) Show that the bounded function f is Integrable on $[a, b]$ if and only if for each $\varepsilon > 0$ there exist $\delta > 0$ and partition P with $\mu(P) < \delta$ and $U(P, f) - L(P, f) < \varepsilon$. **(7)**

b) Show that constant function K is integrable on $[a, b]$ and $\int_a^b k\, dx = k\,(b - a)$. **(4)**

c) Show that the function defined by $f(x) = \begin{cases} 0 \; if \; x \; is \; rational \\ 1 \; if \; x \; is \; irrational \end{cases}$ is not integrable on any interval . **(3)**

Q-8 **Attempt all questions** **(14)**

a) If f_1 and f_2 are two integrable functions on [a , b].show that $f_1 + f_2$ is also integrable on [a ,b] further $\int_a^b (f_1 + f_2)\, dx = \int_a^b f_1\, dx + \int_a^b f_2\, dx$ **(6)**

b) State and prove necessary and sufficient condition for integrability of bounded function on [a , b]. **(4)**

c) Show that 3x+1 is integrable on [1,2] and $\int_1^2 (3x+1) dx = \frac{11}{2}$. **(4)**